MATH122

END SEMESTER EXAMINATION: APRIL–MAY, 2016

APPLIED MATHEMATICS – II

Time : 3 Hrs. Maximum Marks : 70

Note: Attempt questions from all sections as directed.

SECTION – A (30 Marks)

Attempt any five questions out of six.
Each question carries 06 marks.

1. Solve \( x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 2y = x \log x \).

2. Form a partial differential equation by eliminating the arbitrary function from \( \phi(x + y + z, x^2 + y^2 - z^2) = 0 \).

3. Use Cauchy’s Integral formula to evaluate

\[
\int_{C} \frac{\cos^2 z}{(z - \frac{\pi}{2})^3} \, dz \text{ where, } C \text{ is the circle } |z| = 1.
\]

P.T.O.

(1476)
4. Find the general solution of

\[ x \left( z^2 - y^2 \right) \frac{\partial z}{\partial x} + y \left( x^2 - z^2 \right) \frac{\partial z}{\partial y} = z \left( y^2 - x^2 \right). \]

5. Use Contour Integration to evaluate the real integral

\[ \int_{0}^{\infty} \frac{dx}{\left( 1 + x^2 \right)^3}. \]

6. Solve \( \frac{dx}{dt} + 2y + x = e^{t} \)

\[ \frac{dy}{dt} + 2x + y = 3e^{t} \]

SECTION – B (20 Marks)

Attempt any two questions out of three.
Each question carries 10 marks.

7. (a) Solve \((1+xy)xdy + (1-xy)y \, dx = 0.\) \hspace{1cm} (5)

(b) Determine the poles of the function

\[ f(z) = \frac{z^2 - 2z}{(z-1)(z^2 + 1)} \]

and then find the residue at each pole. \hspace{1cm} (5)

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8. (a) Solve \((p^{2} + q^{2})y = qz.\) (5)

(b) If \(f(z)\) is an analytic function with constant modulus, show that \(f(z)\) is constant. (5)

9. (a) Find the image of the infinite strip \(\frac{1}{4} \leq y \leq \frac{1}{2}\) under the transformation \(w = \frac{1}{z}.\) (5)

(b) Solve \(x^{2}r - 3xys + 2y^{2}t + px + 2qy = x + 2y.\) (5)

SECTION - C (Compulsory) (20 Marks)

10. (a) Solve \((D^{2} + a^{2})y = \tan(ax).\) (6)

(b) If \((a_{1} + ib_{1})(a_{2} + ib_{2}) \ldots (a_{n} + ib_{n}) = A + iB,\) prove that

\[(\text{i}) \quad (a_{1}^{2} + b_{1}^{2})(a_{2}^{2} + b_{2}^{2}) \ldots (a_{n}^{2} + b_{n}^{2}) = A^{2} + B^{2}\]

\[(\text{ii}) \quad \tan^{-1}\left(\frac{b_{1}}{a_{1}}\right) + \tan^{-1}\left(\frac{b_{2}}{a_{2}}\right) + \ldots + \tan^{-1}\left(\frac{b_{n}}{a_{n}}\right) = \tan^{-1}\left(\frac{B}{A}\right)\]

(8)

P.T.O.

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(c) Solve \( \frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = x + y \).